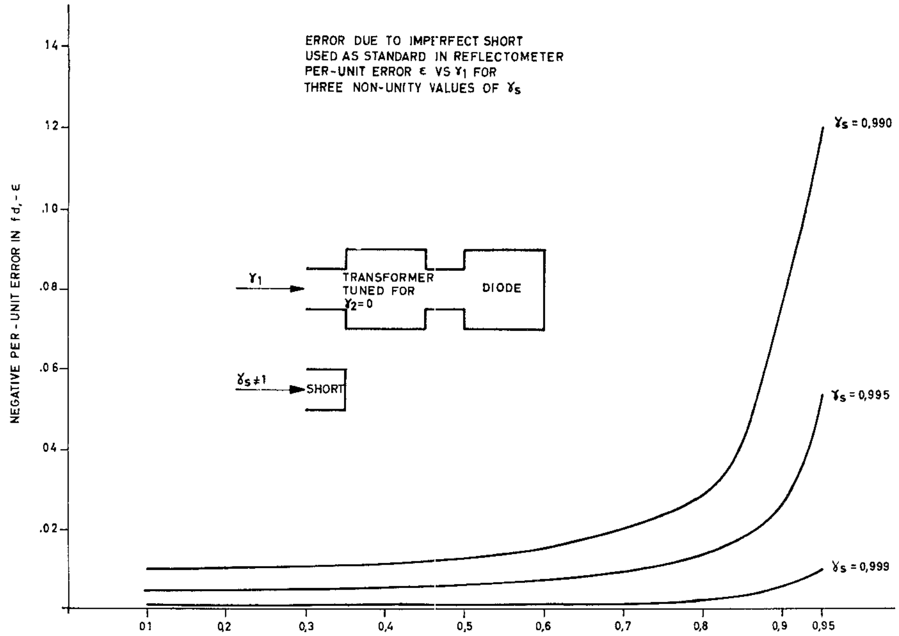

Fig. 3. Measured input reflection coefficient $\gamma_1 = |\Gamma_1|$.

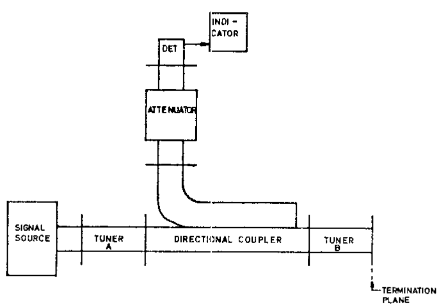

Fig. 4. Basic components of three-port reflectometer.

If this is accomplished, the error in f_d should be 5 per cent or less for values of γ_1 of 0.85 or less.

The simplicity of (7) and the subsequent error analysis are based on the assumption that the ratio γ_s/γ_1 can easily be measured as R dB. This measurement can be performed accurately with a three-port reflectometer [12], [13], shown in block form in Fig. 4. Tuners A and B are adjusted so that a match is seen looking into the termination plane and the directional coupler has nearly infinite directivity. Under these conditions, the signal incident on any load connected to the termination plane is constant. When two loads of different γ are successively connected to the termination plane, the attenuator setting must be changed by R dB to keep the detector signal constant; the ratio of the two γ is determined from (6).

The quality parameter f_d is then measured at frequency f by adjusting a well-made transformer ahead of a diode mount so that $\gamma_2 = 0$, obtaining a detector reference signal for the conditions of γ_1, and recording the differential attenuator reading R necessary to maintain this signal when the diode-mount-plus-transformer combination is replaced by a known load.

M. M. Brady
Norwegian Defence Research Estab.
Kjeller, Norway

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The Characteristic Impedance and Velocity Ratio of Dielectric-Supported Strip Line

In its most practical form, strip line is made with a center conductor which consists of two thin strips of copper of the desired width on each of the two faces of a printed circuit board. Without exception known to the authors, all published theoretical results for the characteristic impedances of such lines involve the neglect of the dielectric board, i.e., the configuration analyzed is that having a center conductor of two thin unsupported strips [1].[1] The only data published to date in which the effect of the supporting card is included have been experimental [2].

In an earlier communication [3] one of the authors outlined an IBM 7090 computer program which was then being developed for the numerical analysis of TEM mode transmission lines by a finite difference approach. This program, now much enlarged and considerably accelerated, has been applied to the solution of this problem.

Theory

The strip-line configuration that was analyzed is shown in Fig. 1. The supporting card used is rexolite 2200, which has a published dielectric constant of 2.65. The object is to determine the strip width required to give a characteristic impedance of 50 ohms and the corresponding velocity ratio. The assumption is made that the system is lossless.

The relevant theory is as follows. For a selected strip width, let the capacity per unit length be computed assuming the dielectric to be absent; this is denoted by C_0. Now let the card be introduced and the capacity C per unit length recomputed. Since the inductance per unit length is clearly not changed by the presence of the dielectric, it follows that

$$Z_0 = 1/v\sqrt{CC_0} \qquad (1)$$

$$v/v_0 = \sqrt{C_0/C} \qquad (2)$$

where v is the phase velocity in the line, and v_0 is the phase velocity of light in free space.

For this work, strip widths increasing in 1/32-inch steps from 7/32 to 11/32 inch were used; the computed characteristic impedances and velocity ratios are shown in Fig. 2. From these it is concluded that a strip width of 0.279 inch must be used to obtain a 50-ohm line, and that the corresponding velocity ratio is 0.9367. From the general closeness of the velocity ratio to unity, it is seen—as would be expected—that the dielectric has a small, though significant, effect.

To test the effect of changing the dielectric material, the calculations for the 9/32-inch strip width were repeated with a dielectric constant of 2.72 (about a 3 per cent increase). The results, as summarized in

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[1] Since submitting this material the authors' attention has been called to the following paper. Foster, K. The characteristic impedance and phase velocity of high Q triplate line, *J. Brit. IRE*, Dec 1958, pp 715–723.

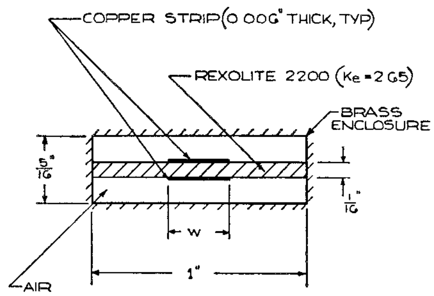

Fig. 1. Basic shielded strip-line configuration.

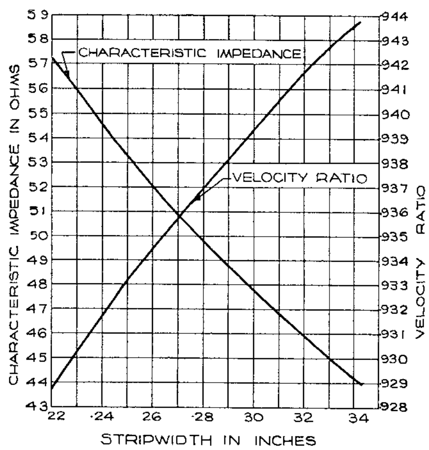

Fig. 2. Computed characteristic impedance and velocity ratio for strip-line section.

TABLE I

Effect of Varying Dielectric Constant

Dielectric constant	Characteristic impedance	Velocity ratio
2.65	49.74	0.9372
2.72	49.63	0.9350
Percentage variation	0.22	0.24

Table 1, show that the variation is not great.

Accuracy

The question of the accuracy of these results is obviously important. As stated in the earlier note [3], this method can be expected to give capacity values which are too high—how much too high can be estimated by comparing the answers for two thin unsupported strips obtained from the finite difference solution with those obtained from Cohn's conformal transformation solution [1]. This solution itself involves an approximation (designed to remove one vertex from the path of integration, thereby reducing the problem to one involving elliptic integrals of the first kind only), but this is thought to be within about 0.1 per cent for the conductor configuration considered here. For a strip of width 9/32 inch Cohn's formula gives a capacity per meter of 62.46 pF. This is to be compared with a result of 62.84 pF/m from the finite difference solution; an error of about 0.6 per cent is evident.

The use of Cohn's formula as a check also involves the neglect of the effect of the end walls. His expression assumes that the top and bottom walls of the enclosure are of infinite extent. However, the width of the physical enclosure is sufficient in proportion both to the width of the center strip and its spacing from the top and bottom walls that the effect of the end walls should be small [4]. What perturbation they introduce will be to increase the capacity above the Cohn value and, thereby, exaggerate the apparent error; i.e., 0.6 per cent is a pessimistic estimate.

It would be expected that the error would not be significantly altered when the card is present so that this is the order of the error to be expected in the computed characteristic impedances. The error in the velocity ratio should be much smaller since this involves the quotient of two nearly equal quantities having approximately the same error.

Examination of Fig. 2 reveals that in the vicinity of 50 ohms the impedance vs width graph has a gradient of about 1 ohm per 0.009 inch, which is closely the amount by which the impedance of a parallel plate line would have changed for the same change in conductor width. This results from the fact that the strip width is sufficiently great that when altered over the range considered in this work the net effect is closely equivalent to removing a section of parallel plate line, i.e., the fringing fields at each end are substantially unchanged.

Experimental Verification

An experimental determination of the characteristic impedance of rexolite-supported strip line ($t/b = 0.2$) must be influenced by the coaxial-to-strip-line transition used. An earlier communication [5] described a broadband coaxial-to-strip-line transition, and a strip width of 0.270 inch gave the optimum match to a 50-ohm coaxial slotted line.

It may be shown that the strip-line impedance must be slightly greater than 50 ohms in order to give a good match when seen through the low-pass network representing the compensated butt transition. This means that the actual strip width for 50-ohm rexolite-supported strip line ($t/b = 0.2$) is slightly greater than 0.270 inch (which agrees with the foregoing theoretical evaluation).

The phase constant of rexolite-supported strip line may be determined by the method illustrated in Fig. 3. Successive known lengths, d, of strip line are removed without disturbing the coaxial-to-strip-line transition. The variation, x, in the position of a minimum in the coaxial slotted line may be used to calculate the ratio of the strip-line wavelength, λ_s, to the coaxial line wavelength λ_0.

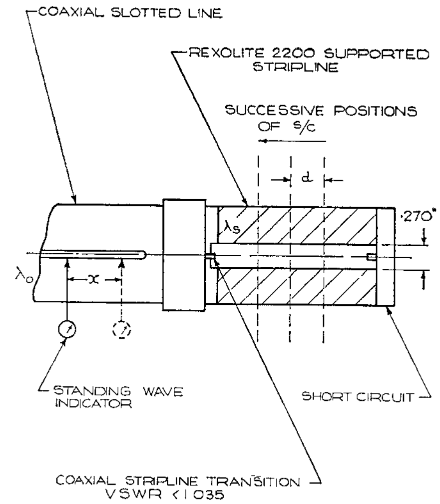

Fig. 3. Determination of the phase constant of rexolite supported strip line.

If d is approximately equal to λ_s, then it may be shown that

$$\frac{\lambda_s}{\lambda_0} = \frac{v}{v_0} = \frac{d}{\lambda_0 + x}$$

Fifteen determinations of λ_s/λ_0 were carried out at each of three frequencies chosen such that x was negative, approximately zero, and positive. The average of 45 determinations was taken as the experimental value, viz.,

$$v/v_0 = 0.933 \pm 0.003$$

Conclusions

Good agreement has been found between computed and experimentally determined values for the characteristic impedance and phase velocity of rexolite-supported strip line. In the case of the phase velocity determination, the computed result is just above the upper limit of the experimental determination. This may be due, in part, to errors in the assumed dielectric constant (see Table I) and in neglecting the strip thickness.

The importance of knowing accurate values for the characteristic impedance and phase velocity of dielectric-supported strip line is demonstrated in the design [6] of capacitive gap band-pass filters. Deviations from the design (based on λ_0) center frequency in experimental filters have been noticed, and these are commensurate with the phase velocity determination already discussed.

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H. E. GREEN
J. R. PYLE
Applied Physics Div.
Weapons Research Estab.
Salisbury, Australia

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Radiation from Rectangular Waveguide with Ferrite Slabs

The problem of radiation from a rectangular waveguide completely filled with transversely magnetized ferrite has been discussed by Tyras and Held [1]. The fundamental TE mode in a rectangular waveguide containing two transversely magnetized symmetrical slabs placed against the side walls has been discussed by Lax and Button [2]–[4], and numerous references to this subject have been made by several other authors [5]–[7]. In the present communication we consider the radiation from the open end of a rectangular waveguide with two ferrite slabs of different thickness placed against the side walls, magnetized by two different static transverse magnetic fields, as shown in Fig. 1. Assuming harmonic time variation $e^{j\omega t}$, the electric fields for the fundamental TE mode are given by

$$E_y = A \sin k_1 \left(\frac{a}{2} + x\right) e^{-j\gamma z} \quad \text{Region I}$$

$$E_y = B \sin k_2 \left(\frac{a}{2} - x\right) e^{-j\gamma z} \quad \text{Region II}$$

$$E_y = \left[C \sin k_3 \left(\frac{a}{2} + x\right) + D \cos k_3 \left(\frac{a}{2} + x\right)\right] \cdot e^{-j\gamma z} \quad \text{Region III} \quad (1)$$

where γ is the propagation constant and

$$k_1^2 = \omega^2 \epsilon_1 \mu_{e1} - \gamma^2 \quad \text{Region I}$$
$$k_2^2 = \omega^2 \epsilon_2 \mu_{e2} - \gamma^2 \quad \text{Region II}$$
$$k_3^2 = \omega^2 \epsilon_0 \mu_0 - \gamma^2 = k^2 - \gamma^2 \quad \text{Region III} \quad (2)$$

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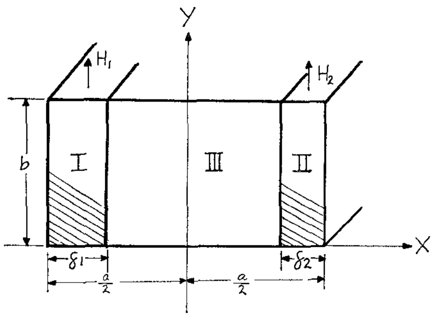

Fig. 1. Waveguide aperture.

where

$$\mu_{e1} = \frac{\mu_1^2 - K_1^2}{\mu_1}; \qquad \mu_{e2} = \frac{\mu_2^2 - K_2^2}{\mu_2}$$

By matching E_y and H_z at the appropriate boundaries, one obtains

$$a = \delta_1 + \delta_2 + \frac{1}{k_3} \tan^{-1} \frac{N_1 \sin k_2\delta_2 - N_2 \sin k_1\delta_1}{\sin k_1\delta_1 \sin k_2\delta_2 + N_1 N_2} \quad (3)$$

where

$$N_1 = \frac{\mu_0}{k_3}\left[\frac{K_1\gamma}{\mu_1\mu_{e1}} \sin k_1\delta_1 + \frac{k_1}{\mu_{e1}} \cos k_1\delta_1\right]$$

$$N_2 = \frac{\mu_0}{k_3}\left[\frac{K_2\gamma}{\mu_2\mu_{e2}} \sin k_2\delta_2 - \frac{k_2}{\mu_{e2}} \cos k_2\delta_2\right]$$

By substituting (2) into (3) one obtains a transcendental equation for the propagation constant γ. For the particular cases of identical slabs ($\delta_1 = \delta_2$; $k_1 = k_2$) and a single slab against the side wall ($\delta_2 = 0$), (3) becomes similar to the results obtained originally by Lax and Button [2].

The magnetic field component H_x can readily be obtained [6] from (1) to give

$$H_x = -\frac{A}{\omega\mu_{e1}}\left[\gamma \sin k_1\left(\frac{a}{2} + x\right) + \frac{K_1 k_1}{\mu_1} \cdot \cos k_1\left(\frac{a}{2} + x\right)\right] \quad \text{Region I}$$

$$H_x = -\frac{B}{\omega\mu_{e2}}\left[\gamma \sin k_2\left(\frac{a}{2} - x\right) - \frac{K_2 k_2}{\mu_2} \cdot \cos k_2\left(\frac{a}{2} - x\right)\right] \quad \text{Region II}$$

$$H_x = -\frac{\gamma}{\omega\mu_0}\left[C \sin k_3\left(\frac{a}{2} + x\right) + D \cdot \cos k_3\left(\frac{a}{2} + x\right)\right] \quad \text{Region III} \quad (4)$$

where the common factor $e^{j(\omega t - \gamma z)}$ is understood.

From these relationships, derived by matching E_y and H_z at the boundaries, one may also obtain

$$B = \frac{A}{\sin k_2\delta_2}\left[\sin k_1\delta_1 \cos k_3(a - \delta_1 - \delta_2) + N_1 \sin k_3(a - \delta_1 - \delta_2)\right]$$

$$C = A\left[\sin k_1\delta_1 \sin k_3\delta_1 + N_1 \cos k_3\delta_1\right]$$

$$D = A\left[\sin k_1\delta_1 \cos k_3\delta_1 - N_1 \sin k_3\delta_1\right] \quad (5)$$

In order to calculate the far-zone radiation fields, it is assumed [1] that the field distribution at the open end of the waveguide is the same as if the waveguide were infinite in extent. The far-zone radiation fields will be evaluated from the relationships given by Silver [8]. In the H plane ($\phi = 0$) one has

$$E_\phi = \frac{jk}{4\pi R} e^{-jkR} \iint_A (E_y \cos\theta - \eta H_x) \cdot e^{jkx \sin\theta}\, dxdy \quad (6)$$

and in the E plane ($\phi = \pi/2$) one has

$$E_\theta = \frac{jk}{4\pi R} e^{-jkR} \iint_A (E_y - \eta H_x \cos\theta) \cdot e^{jky \sin\theta}\, dxdy \quad (7)$$

where

$$k = \omega\sqrt{\mu_0\epsilon_0} \quad \text{and} \quad \eta = \sqrt{\mu_0/\epsilon_0}$$

The calculation is carried out for each region separately, the final result being the superposition of the three radiation fields. Substituting (1) and (4) into (6), one obtains for the H plane ($\phi = 0$) $E_\theta = 0$, and E_ϕ for region I is

$$E_\phi = \frac{jAbk}{4\pi} \frac{e^{-jk(R - a/2 \sin\theta)}}{R(k_1^2 - k^2 \sin^2\theta)} \cdot [k_1 \cos\theta + A_1 \sin\theta + A_2 + e^{jk\delta_1 \sin\theta}(A_3 \sin\theta\cos\theta + A_4 \cos\theta + A_5 \sin\theta + A_6)] \quad (8)$$

where

$$A_1 = -jk\frac{\eta k_1 K_1}{\omega\mu_{e1}\mu_1}; \qquad A_2 = \frac{\eta k_1 \gamma}{\omega\mu_{e1}}$$

$$A_3 = jk \sin k_1\delta_1; \qquad A_4 = -k_1 \cos k_1\delta_1$$

$$A_5 = \frac{jk\eta}{\omega\mu_{e1}}\left(\gamma \sin k_1\delta_1 + \frac{k_1 K_1}{\mu_1} \cos k_1\delta_1\right)$$

$$A_6 = -\frac{\eta k_1}{\omega\mu_{e1}}\left(-\frac{k_1 K_1}{\mu_1} \sin k_1\delta_1 + \gamma \cos k_1\delta_1\right)$$

A similar result is obtained for region II, where one should use (8), taking B, k_2, K_2, μ_{e2} instead of A, k_1, K_1, μ_{e1}, and ($-\delta_2$) instead of δ_1. The results for region III may be found in Chien [9].

Substituting (1) and (4) into (7), one obtains for the E plane ($\phi = \pi/2$) $E_\phi = 0$, and E_θ for region I is

$$E_\theta = \frac{A}{2\pi k_1 R} \frac{\sin(\frac{1}{2}bk \sin\theta)}{\sin\theta} \cdot e^{-jkR}[D_1 \cos\theta + D_2] \quad (9)$$

where

$$D_1 = \frac{\eta}{\omega\mu_{e1}}\left[\gamma(1 - \cos k_1\delta_1) + \frac{k_1 K_1}{\mu_1} \sin k_1\delta_1\right]$$

$$D_2 = 1 - \cos k_1\delta_1$$

A similar result is obtained for region II by use of the transformation already de-